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SURVEY OF STRUCTURAL SAFETY

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1.0 INTRODUCTION

The design of any structure is dependent on the individual study of a "load" characteristic and a "strength" property, and the adequacy of the design is generally assured by maintaining a "factor of safety" between such a load and strength. Ordinarily these three essential elements for design, i.e., the load, the strength, and the safety factor, are the responsibility of certain specification writing bodies, the only responsibility of the designer and structures engineer being to meet the criteria specified by such groups.[1] Thus, the actual safety of the final structure depends, first of all, on the adequacy of the design criteria, and secondly, on the ability of the product to meet specifications.

As is well known, considerable effort has been expended over the past half-century towards improving our understanding of the behavior of structures and structural materials. Great progress has been realised in methods of stress analysis and more efficient materials have been introduced. Unfortunately, however, the design criteria phase of the problem has been relatively ignored, especially the role of safety factors. Only recently has it been realised that, <u>no matter how conservative a design may be, there exists a real chance of its failure, and that such a failure probability must form the basis for any rational design procedure</u>. The use of rather arbitrary constant multipliers, as factors of safety, results in a design which may be highly unbalanced with respect to safety, and which has no clearly discernible probability of failure. It is senseless to employ highly refined methods of strength analysis unless equal consideration is also given to the load analysis and to the safety margin between strength and load.

1. Structural design criteria for aero-space vehicles and GSE are prepared by the contractor's structures group. These structural design criteria reports define loading conditions, establish or reference factors of safety, list approved specifications, sources, etc., and are submitted to the contracting agency for approval.

With the realization that current design procedures lack a rational basis for safety, several questions naturally come to mind: What really is the significance of a safety factor? What practical benefits can be derived from attempting to establish more realistic measures of safety? What factors influence the probability of failure? What probability of failure is realistically acceptable? What is the statistical nature of loads and environmental conditions? - of strength of materials and behavior of structures? And finally: What steps must be taken to provide more rational bases for design decisions?

In the past dozen-or-so years a flourish of writings has appeared endeavoring to answer such questions. Increasing emphasis on these problems is apparent in both the civil engineering and aeronautical engineering fields, as can be seen from the bibliography at the end of this report. Of particular significance in the field of civil engineering is the recent organization of the Committee on Factors of Safety by the American Society of Civil Engineers (ref. I-37, 53, 62). The purpose of this committee is to define factors of safety in relation to probabilities of failure or unserviceability and to recommend forms of such factors for future use. The practical benefits possible through these studies are quite obvious. Not only can the reliability of structures be controlled, but savings in weight and cost, and increased performance, can be realized without sacrificing safety.

The intent of this report is to summarize the fundamental concepts of structural safety and review the progress being made towards understanding its true nature. The probability of failure is introduced as a structural design criterion, and the role of safety factors in design is discussed in relation to probability concepts. The variable nature of loads and material properties is only briefly discussed as it will be the subject of later reports. Throughout this report the terms "loads", "strength", and "failure" are used in their broadest sense. By "load" is meant

any imposed condition of forces, temperature, vibration, deformation, etc; by "strength" is meant the capacity of a structure to resist such "loads"; and "failure" implies any undesirable condition of fracture, yielding, wear, creep, etc.

2.0 ENGINEERING DECISION PROCESSES

In order to understand the problem of structural safety it is well to first consider the basic logic behind decision making processes. The fundamental difficulty in engineering is that conclusions must be drawn about a future situation the exact nature of which is unknown. Thus, the real problem is one of prediction of future behavior and events and, as such, is not explicitly solvable. In making a decision about a real situation the engineer or scientist must resort to abstractions of the real problem and base his conclusions on the interpretation of abstract results. Such a decision process is shown schematically in Figure 1.

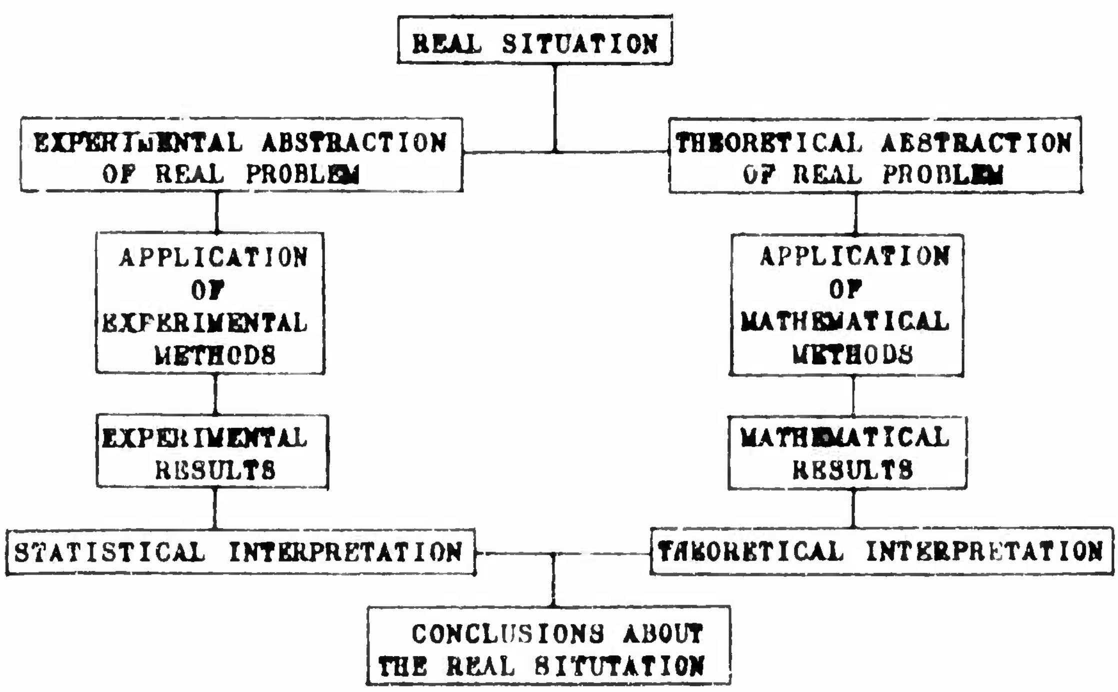

Figure 1: Schematic Representation of an Engineering Decision Process (ref. IV - 10)

The flow diagram illustrated in Figure 1 shows that conclusions about a real world problem may be reached by two fundamentally different paths, the one based on observations of actual past events, and the other on the laws of abstract science. This is, of course, a simplified representation, as each path usually involves some concepts from the other, and each operation may include many separate decision processes. By following both paths simultaneously and coordinating results, the reliability of the final conclusions can often be increased. Another, more common, procedure for increasing confidence is to run several analyses in series, using the results of each preceding investigation to obtain a better abstract model for the next analysis. Such a continual refinement process has been the usual course of events throughout the history of science and engineering.

The important thing to realize is that the solution to an engineering problem is not as clear-cut as we would like to think. In fact, the exact nature of the problem itself is unknown since we are ignorant of future reality. We can only deal with an abstraction of the real problem, apply experimental logic and/or mathematical logic to the abstract model, and use the abstract results as a guide to decision making. Such a process always involves a certain amount of judgement. Neither mathematical deductions nor statistical inferences can provide us with a real solution, but both can become effective tools which, if properly used, will lead us to more rational conclusions.

3.0 THE PROBABILITY OF FAILURE AS A STRUCTURAL DESIGN CRITERION

In establishing criteria on which to base a structural design we are required to define conditions of "load" and "strength" (ref. pages 3, 4). We therefore must predict realistic levels of these conditions; generally, the extreme levels which our structure may possibly encounter within its lifetime. Since it is impossible to predict such conditions with certainty, we must obtain rational estimates through the use of the logical decision processes previously discussed. In doing this we accept a certain degree of risk that our conclusions are incorrect and, consequently, there will always exist a definite probability of failure. It is logical that, as our first step in design, we define an acceptable maximum value for this probability of failure.

The level of failure probability to which structural component should be designed must be determined by weighing its functional importance and economic value against the consequences and cost of failure. As previously mentioned, there are several types of structural failure conditions to consider. Catastrophic[1]

1. In order to establish and define the critical condition of <u>catastrophic failure</u> of a structure, the different modes of collapse of structural resistance must be considered. Collapse can be produced by:

   a. The instability or fracture of one or several primary structural elements or connections under a single application of an excessive load or load-temperature condition.

   b. The fracture of one or several primary structural elements or connections as a result of creep or fatigue under the sustained or intermittent application of a random (or periodic) sequence of loads and temperatures, the intensity and frequency of which is described by a three dimensional load-temperature-time spectrum.

as well as minor[2] failures may result from instability, fracture, or fatigue. Unserviceability will be caused by such conditions as yielding, excessive creep deformation, insufficient stiffness, wear, and corrosion. It is apparent that some of these failure conditions are more important than others. We can accept, for example, a greater failure probability for yielding of a structure then for catastrophic fatigue. Thus we have to consider each type of failure and unserviceability condition separately and evaluate the different risks involved in order to arrive at design levels of failure probability.

The probability of failure of aircraft or missile structures may be expressed in various forms (ref. I - 35, 39, 43, 44, 51). For example, we may express the failure rate of an aircraft as the expected failures per mile of flight, per hour of flight, or per mission. Typical design values of tolerable failure probabilities of civil aircraft, expressed in hours of flight, would be of the order of magnitude of 10^{-4} for yielding, 10^{-7} for failures under ground-load conditions, 10^{-8} for in-flight failures other than fatigue, and 10^{-9} for catastrophic fatigue failures (ref. I - 44).[3]

Another form for specifying failure probability, suggested by Freudenthal (ref. I - 35), is to define the acceptable risk of

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2. Minor failures would result from similar modes of collapse of one or several secondary structural elements or connections.

3. One should not assume that an, across-the-board, increase in the allowable probabilities of failure for missiles is advisable; the contrary may be true, "It should be remembered that components used in guided missiles must be more reliable than those of piloted aircraft by about one order of magnitude" (ref. I - 11). A thorough statistical investigation along a missile life-mission concept approach and accounting for considerations of safety (ref. p. 11) is required to determine allowable probabilities of failure.

failure over the entire service lifetime of the structure. Taking the above example of an aircraft failure rate of 10^{-8} per hour, and supposing that the airplane is expected to have a service life of 20,000 hours, then the chance of an aircraft ever having an in-flight structural failure in its lifetime would be 1 in 5000. This method of expressing failure risk in relation to the overall life of the structure seems to provide the simplest and most rational basis for a design criteria. It is expecially well-suited for missile structures.

In connection with lifetime failure probabilities, Freudenthal defines the following two terms (ref. I - 35):

<u>Return period</u> - the expected time between occurences of an extreme high load - low strength combination which, upon its single application, causes structural failure.

<u>Return number</u> - the number of repetitions or cycles of a standard load pattern that will result in a structural failure.

Since the probability should be extremely small that a structure will, in its lifetime, encounter a load-strength combination that causes failure, the "return period" of such a load-strength combination should be very much longer than the design life of the structure. In the previous example, for instance, the "return period" of the critical load-strength combination that causes in-flight structural failure is 5000 times the design life of the aircraft. Similarly, the "return number" of a load pattern should be made very much higher than the total number of load applications expected during the lifetime of the structure.

4.0 THE SIGNIFICANCE OF SAFETY FACTORS

In aircraft and missile fields the current practice is to employ the "limit load" design concept, as opposed to the "working stress" approach used in mechanical and civil engineering (these two concepts are discussed by Goldin in ref. I - 38). Only the "limit load" type of criteria will be considered in this report.

The term "limit" (as applied to aircraft structural design criteria) first appeared in Part 04 of the Civil Air Regulations of 1938. It was used to specify the actual maximum load factors expected to be experienced by aircraft in given flight and ground handling conditions. As such, "limit load factors represented actual "limiting" conditions of acceleration of an aircraft, the factors themselves being the ratio between loads in the accelerated and unaccelerated (1g) conditions. The term "limit" was also adopted by the Army and Navy through the efforts of the ANC Committee on Aircraft Requirements (a history of the development of aircraft design criteria is given by Mangurian in ref. I - 21). Current Civil Air Regulations (Parts 3 and 4b) define limit load as "the maximum load anticipated in service". Similar definitions appear in corresponding Air Force and Navy specifications for aircraft and missiles. Design criteria based on the limit load philosophy have proved to be adequate for flight structures of the past. However, with the advent of missiles and high speed aircraft, and with the use of new materials at high working stresses, consideration must be given, not just to maximum load conditions; but, to the entire load-temperature-time history of the vehicle. "Limit conditions", defining load pattern, cycles of loading, temperature, and time durations, should replace the "limit load" concept in order to effectively account for aerodynamic heating and fatigue problems in design (ref. I - 38, 39).

Since limit loads, or rather limit conditions, represent actual situations likely to be encountered,[1] certain factors of safety are required to ensure against failure or unserviceability of the structure under these conditions. Such safety factors must account for all of the following:

1. The accuracy of predicted loads and environment.[2]

2. The degree of variability and the nature of the frequency distributions of the design loading conditions.

3. The accuracy and extent of the stress analysis, fatigue analysis and/or the degree of experimentation.

4. The variability in the resistance of materials and structures.

5. The degree of inspection and quality control.

6. The variability of residual stresses or eccentricities resulting from tolerance build-up, misalignment, etc., due to poor design, material control or assembly.

7. The degree of maintenance of the original strength standard (effects of deterioration due to corrosion or deficient maintenance) by the operators during the life of the vehicle.

8. The degree of workmanship, tolerance limits and surface finish specified for the manufacture of structural elements.

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1. Within the calculated probability of their occurrence.

2. Loads and environment for launch and flight conditions for aero-space vehicles are usually predicted with sufficient accuracy so that the reduced factor of safety for these vehicles does not account for any load or environment errors or uncertainties.

9. The estimated value of the structure.[3]

3. The value of all the benefits which the structure can be expected to provide for its users, can be expressed as a capital sum which we might call the service value of the structure. It is the engineer's task to design the structure so that its service value exceeds, by as much as possible, the cost of producing and maintaining it. This production and maintenance cost is made up of the original cost of construction, the capitalised cost of service and maintenance during its service life and the cost of repairing or reconstructing the whole structure or its separate parts because of deterioration, failure or collapse. In the cost of such mishaps must be included the cost of damage to means of production other than the structure itself (other structures, vehicles, etc.) and the costs of injury or loss of life.

5.0 THE EFFECTS ON STRUCTURAL DESIGN OF A PROBABILITY OF FAILURE

FACTOR OF SAFETY APPROACH

At present, the engineer is faced with the joint effect of uncertainties in external loadings and internal strength of materials. The importance of a correlation of these two random phenomena cannot be emphasized enough, and a reason why this has been neglected so long is that there are usually two entirely different groups of specialists who formulate or influence design loads and minimum strength values: There are government officials or technical groups, on one side, while on the other side are material manufacturers, government material specification writers and specialists in the technology of materials. (Most aero-space vehicle manufaturers are attempting to close this gap in structural design technology.) Clearly, it is the structures engineer who stands between the two groups and who has to bring the two sources of information together by means of the structural analysis. If, however, the structures engineer asks for information from the neighboring branches of engineering in a greatly simplified form (e.g., furnish a single constant for what in reality is a whole distribution function), the two neighboring groups are forced to round their figures off and put a certain safety margin into their specifications on account of the later oversimplified treatment. If, however, a certain safety in the load assumptions is already included in the form of an unlikely or infrequent occurrence, the same is done in the field of material technology, and the structures engineer superposes his own safety factor, then it is likely in some cases that the end result is unreasonably safe.

In an effort, therefore, to achieve maximum economy and at the same time ensure adequate safety, the structures engineer

has to insist that all information on the strength properties of materials and the anticipated loads on the structure are given to him in a completely unbiased form and as realistic as possible. However, in most cases, this is only possible by means of statistics. The structures engineer subsequently ought to be in a position to read this statistical information and derive results and conclusions therefrom.

When the structures engineer has this statistical information for loads and environment, he should be able to divide certain design aspects into two categories and attack the "factor of safety - probability of failure" approach. This will result in an increased and more balanced level of reliability, reduced costs, reduced weight and adequate safety for our future designs.

In both of the following categories, adequate consideration must be given to the factor of safety requirements (specified on p. 11), serviceability and maintainability.

Category A. - Design aspects wherein a reduction in ultimate factor of safety should not be considered at this time. In fact, an investigation, through a probability of failure approach, may indicate an increase in the "factor of safety" would be required to attain the required level of safety.

Examples in Category A

1. When operational requirements of a new vehicle are not definitely determined, and design maneuvering and ground loads and loading distributions cannot be definitely ascertained within small tolerances'.

2. When positive steps are not taken to prevent exceeding the specified design limit maneuver load factors, inadvertently, due to undesirable low stick force in pounds per g and unduly light control forces in general.

3. When adequate experimental data are not available for use in design, and before delivery of the vehicle.

Examples in Category A (contd.)

4. When structural behavior due to aerodynamic heating or other phenomena cannot be accurately determined.

5. When non-linearity in aerodynamic data or structural deflections can be catastrophic if the limit design conditions are exceeded by only a small amount. This is especially serious, since many people have thought that an ultimate factor of safety of 1.5 indicates that the vehicle strength is good for an ultimate load factor of 50% above the limit load factor.

Category B - Design aspects wherein a reduction in the ultimate factor of safety[1] should be considered and can best be determined through a probability of failure approach. (Structure designed in this category would still have no perceptible set, or yield, at or below the limit loading condition.)

Examples in Category B

(In the following Category B examples, it is assumed that where the term(s) "load(s)", "loading(s)" or "loading condition(s)" have been used, it or they will be based upon the mean or operating value, plus some constant times the standard deviation of the distribution; in order that a true limit value will be formed, having a specified low probability of occurence.)

1. Loadings resulting from ram pressure.

These loadings can be dete[illegible]th fair accuracy and it is not likely that limit design [illegible] will be exceeded if the vehicle stays within its specified speed altitude, trajectory and/or orbit limits. Structures such as intake ducts would be considered under this point.

2. Loadings from pressurization, such as pressurized cabins, personnel enclosures, propellant tanks, etc.

Such loadings are controlled by a pressure relief or control valve and it is not likely that limit design loadings will be exceeded, unless malfunction occurs. It may be more

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1. Where, generally, the ultimate factor of safety now is 1.5, a reduction, to an approximate level of 1.1 - 1.2, should be possible with an adequate statistically based, structural analysis approach.

Examples in Category B (contd.)

economical to install dual relief or control systems rather than to provide excessive strength in the structure for such malfunctions. In any determination of the recommended factor of safety for the bursting of pressure vessels, account should be taken of the difference between real and apparent ultimate factors of safety caused by material property effects (primarily, strain hardening).

3. Loadings from hydraulic systems which have relief valves.

4. Thrust loadings from engines, including rockets.

These loadings are determined with fair accuracy and it is not likely that limit design loads will be exceeded. Generally, a larger percentage variation of the ratio of standard deviation of thrust to mean thrust will be found with solid rockets, than that found with liquid rockets.

5. When loadings are due to buffet boundaries of an air vehicle.

This is a rather questionable item without flight test information, since it is difficult to determine load magnifications when the buffet boundaries are reached or exceeded.

6. When loadings are due to pressure levels approaching absolute vacuum.

7. When loadings are due to terminal velocity.

It is not likely that a limit terminal velocity will be exceeded. Affected structures may include canopies, tail surfaces, inlet ducts, nose cones, re-entry vehicles, etc., depending upon the load distributions.

8. When loadings are due to hinge moment limitations.

Many control surfaces, such as flaps, ailerons, elevons, etc., have hinge moment limitations due to the available force of a servo motor or a hydraulic operating cylinder. Therefore, the maximum available hinge moments on the surfaces can be determined with fair accuracy. If adequate tolerances have been established for center of pressure and center of gravity locations, then the design loads can be determined on the control surface structure and it is not likely that they will be exceeded. In many cases, over-all wing, tail, and fuselage loads and total vehicle load factors are limited to probability based limit levels because of such hinge moment limitations.

Examples in Category B (contd.)

9. When limit loadings result from maximum control surface (or other aerodynamically loaded surface) deflections.

Some design criteria specify maximum control surface (or other aerodynamically loaded surface) deflection, for surfaces such as speed brakes, dive brakes, tabs, cowl flaps, etc., at all design air speeds up to maximum. If good data are obtainable from flight or wind tunnel tests, then it can be stated that the limit loadings should not be exceeded, within the calculated probability.

10. When practical g-limiter or gust alleviator installations are available.

In such cases, it is not likely that the limit loading will be exceeded. However, malfunction of such installations should be taken into consideration. Here again it may be more economical to install a dual system, than to penalize the structural weight to take care of malfunctions.

11. When automatic controls are installed on vehicles for ground or flight conditions and the loading conditions will be restricted to the specified loadings.

This method is already in use on missiles and the criteria of an ultimate factor of safety less than 1.5 has been accepted practice with considerable success. In order for aero-space vehicles of the future to accomplish their missions successfully, it will be necessary to rely more and more on automatic controls rather than manual operations. Probability of failure of such automatic controls may be high, and therefore, dual systems may be required.

12. When the factor of safety times the limit loads encountered from the limit gust velocities result in ultimate gust expectancies considerably in excess of the estimated air life of the vehicle.

13. When the factor of safety times the limit external loads encountered in flight or ground conditions result in ultimate external design loads considerably in excess of ultimate external loads based on the factor of safety times the limit load factors.

14. When stresses are due to aerodynamic heating.

It is not likely that such stresses will be exceeded, if the limit design speed of the vehicle and the rate of temperature rise are within the limit load-temperature-time condition.

Examples in Category B (contd.)

15. When load limits have been determined by adequate flight and/or ground loads demonstrations.

If a vehicle has been subjected to an extensive loads program and it has been demonstrated that certain loading conditions should not be exceeded, then it should be possible to take advantage of these load limitations by formulating new probability based limit loads for design of any future modifications of the vehicle.

6.0 CONCLUSIONS

The concept of 'probability of failure' will not supplant the concept of 'factor of safety' in the entire structural engineering profession for some time. The agencies and the professional people involved are not ready for such a radical change in the engineering approach to structural design and analysis. Enough will be gained if they are gradually reconciled to the fact that the concept of 'factor of safety' is meaningless, unless it is supplemented by the specification of the probability of failure associated with it. Therefore any reasonable, efficient design, even the most complete and conservative one, tacitly implies an accepted risk of failure. The difference between the safe and unsafe design is in the degree of risk considered acceptable, not in the delusion that such a risk can be completely eliminated.

Structural engineering in the aero-space vehicle field, however, is ready for a probability of failure approach to factors of safety and structural analysis; the following conclusions apply to the development of that approach:

1. Correlation of Factor of Safety (or Factor of Serviceability) with probability of survival and probability of serviceability for each individual structural element designed is impractical.

2. It is practical to consider such correlation in framing design rules and regulations. In fact, it is very desirable. It should be inquired, "What is the probability of loss?"

3. Statistical and probability studies are only guides (like mathematical tools) and must be supplemented by the application of common sense and engineering judgment.

4. Work on safety factors will be of little avail until structures engineers have acquired:

   a. A statistical background of information on the resistance of materials and structures, including time-yield, dynamic and fatigue effects.

CONCLUSIONS (contd.)

b. A similar background for load effects.

c. Competence in probability and statistical analyses (involves considerable educational effort).

7.0 RECOMMENDATIONS

This survey pointed out the need for much additional investigative and educational effort.

1. Research and publication of results should be undertaken, as soon as possible, in the following fields:

   a. The variable nature of the resistance of materials and structures.

   b. The variable nature of loads and environment.

2. The education of structures engineers should be fostered so that they may acquire:

   a. A statistical background regarding the resistance of materials and structures, including time-yield, dynamic and fatigue effects.

   b. A similar background regarding load effects.

   c. The necessary competence in the calculus of probability, which includes the elements of statistical analysis.

3. Research leading to recommended factors of safety, and the associated probabilities of failure, for general structural components of aero-space vehicles[1], GHE[2] and GSE[3] should be started, now.

4. Additional statistical data should be collected on loads which occur very rarely, so, as to make possible, a more reliable estimate of the magnitude of standard loads.

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1. Astronautics work, for the present, would be limited to ICBM, satellite and re-entry vehicle components (engines, engine mounts, wings, tails, control surfaces, controls, pressure vessels, hydraulic or pneumatic lines and fittings, propellant tanks, personnel enclosures, etc.).

2. Components of launchers, erection booms, trailers, etc.

3. Components of towers, propellant storage tanks, blockhouses, etc.

RECOMMENDATIONS (contd.)

5. The application of optimum design considerations to structures subjected to two critical load conditions (e.g., a maximum positive load condition and a maximum negative load condition) which affect largely different amounts of material, so as to establish the best distribution of the probabilities of failure between the two individual load conditions and their effect on structural weight.

6. Most mechanical and physcial properties of material used in the construction of aero-space vehicles, GHE and GSE should be evaluated on a probability basis for design allowable properties (methods similar to those presented in ref. IV - 15, should be employed)[4].

7. Tests for determining design allowables of structural elements, critical in instability, or structural connections, critical in fracture, shear, etc., should be evaluated on a statistical basis (methods similar to those presented in ref. IV - 15, should be employed)[4].

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4. Unless the selected probability function is germane to the problem, and adequately represents the inherent statistical variability of the phenomenon, which results from certain basic assumptions concerning its origin, extrapolation toward the extremes (tails of the function) will result in erroneous predictions within this range of variation, which is just the relevant design range.

Preliminary study of the statistical variations of structural design parameters indicate fair correlation, as follows:

Type of Distribution	Structural Application
Normal	Analysis of random loads, loading conditions and material mechanical properties.
Log-normal or Exponential	Analysis of frequency function of gusts, material fatigue properties and material mechanical properties (e.g., F_{tu}, F_{ty}) effected by a good to fair level of quality control.
Gumbel's or Poisson's	Analysis of material mechanical properties (e.g., F_{tu}, F_{ty}) effected by a poor level of quality control.

RECOMMENDATIONS (contd.)

8. All structural element tests conducted to prove structural adequacy or properties should be run with a sample sufficient in size so that realistic statistical conclusions can be drawn.

NOTE: An existing fallacy, in the practical application of statistics, is the use of, mean plus or minus "Three-sigma" values, to estimate the maximum or minimum expected value. This "Three-sigma" procedure is quite common in the estimation of structural design parameters for aero space vehicles.

In the first place, s is meant, not σ. Sigma implies that the entire population is known; which, in most structural design cases, is quite unlikely. The sample standard deviation, s, is the best estimate of σ, the true standard deviation of the entire population.

The fallacy of this "Three-sigma" cook book rule should be obvious. If the initial variate is unlimited, the largest value is unlimited, too, and if the sample size is increased, the largest value encountered will likewise increase. Therefore, for very small sample sizes, the 3s ("Three-sigma") criterion may give an unconservative (not extreme) estimate; for very large sample sizes the 3s ("Three-sigma") criterion may give a too conservative (too extreme) estimate. Some examples of this follow:

Sample Size	Criterion	Probability of Exceeding (with 99% Confidence)	Probability of not Exceeding (99% Confidence)
7	Mean + 3s	22%	78%
11	Mean + 3s	9%	91%
17	Mean + 3s	5%	95%
66	Mean + 3s	1%	99%
∞	Mean + 3σ	0.13%	99.87%

A more scientific and realistic method of estimating maximum or minimum values would be to use extreme value theory and estimate these max or min values to a certain level of probability (e.g. 99% probability, 1% probability, etc.) of occurrence.

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